Comparing Multiple Proportions

Research Objective

Research Question: Is there a relationship between which college someone is in and whether they use an apple or android phone?

Population: All BYU students.

Parameter of Interest:

- We have a lot! We want to know the proportion of students in each college/phone combination. For example, $\pi_{\text{Apple,Humanities}}$.

Sample: A convenience sample of 1727 BYU students who are in my class and completed the student survey.

Are there any issues with this study setup?

More Problem Definitions

Response Variable (y):

- Does the student have an Apple or Android phone. This is a **categorical variable** meaning it has to be one of a certain number of categories.

Explanatory Variable (x):

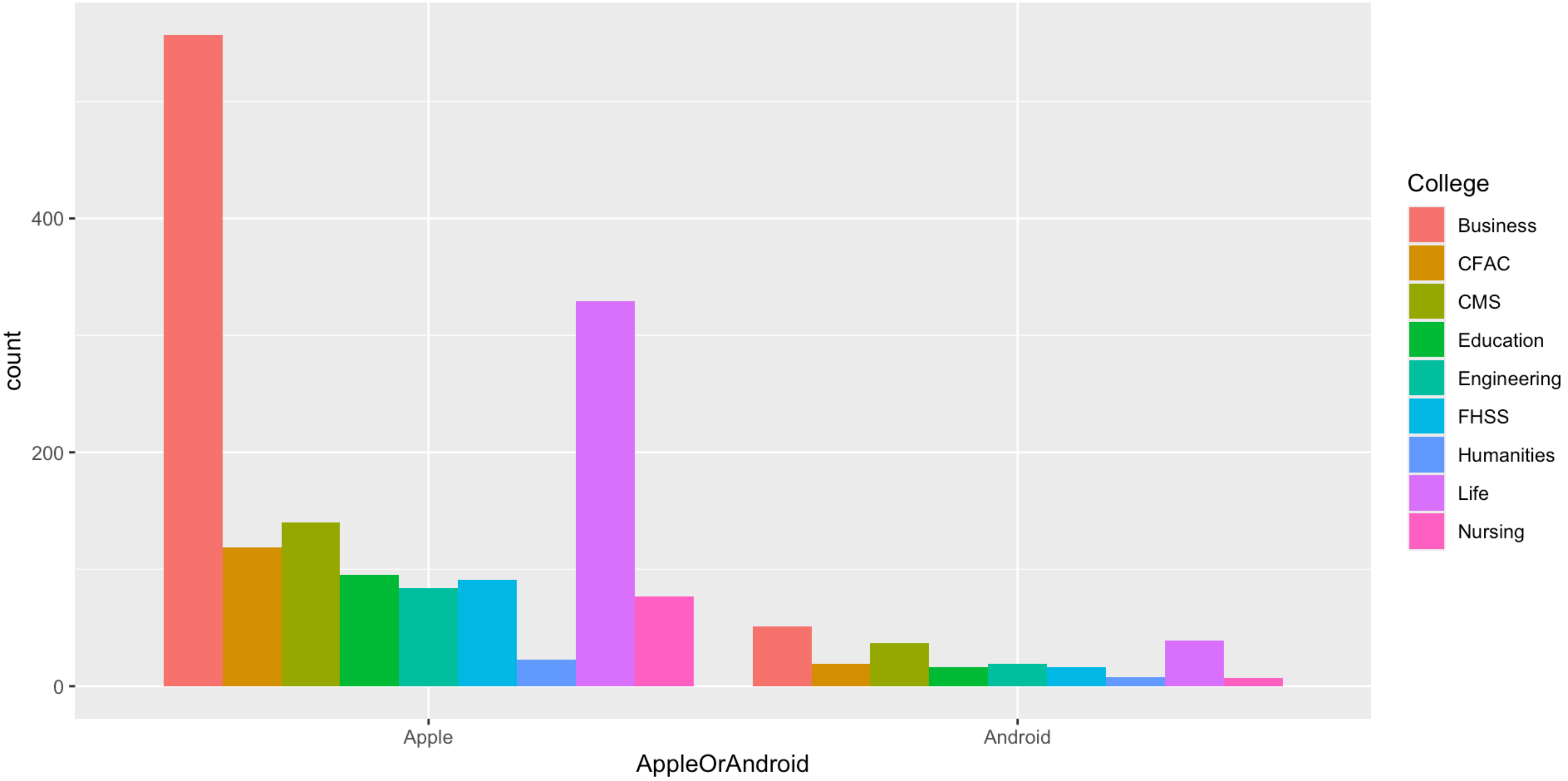
- The college.

Exploratory Data Analysis (EDA)

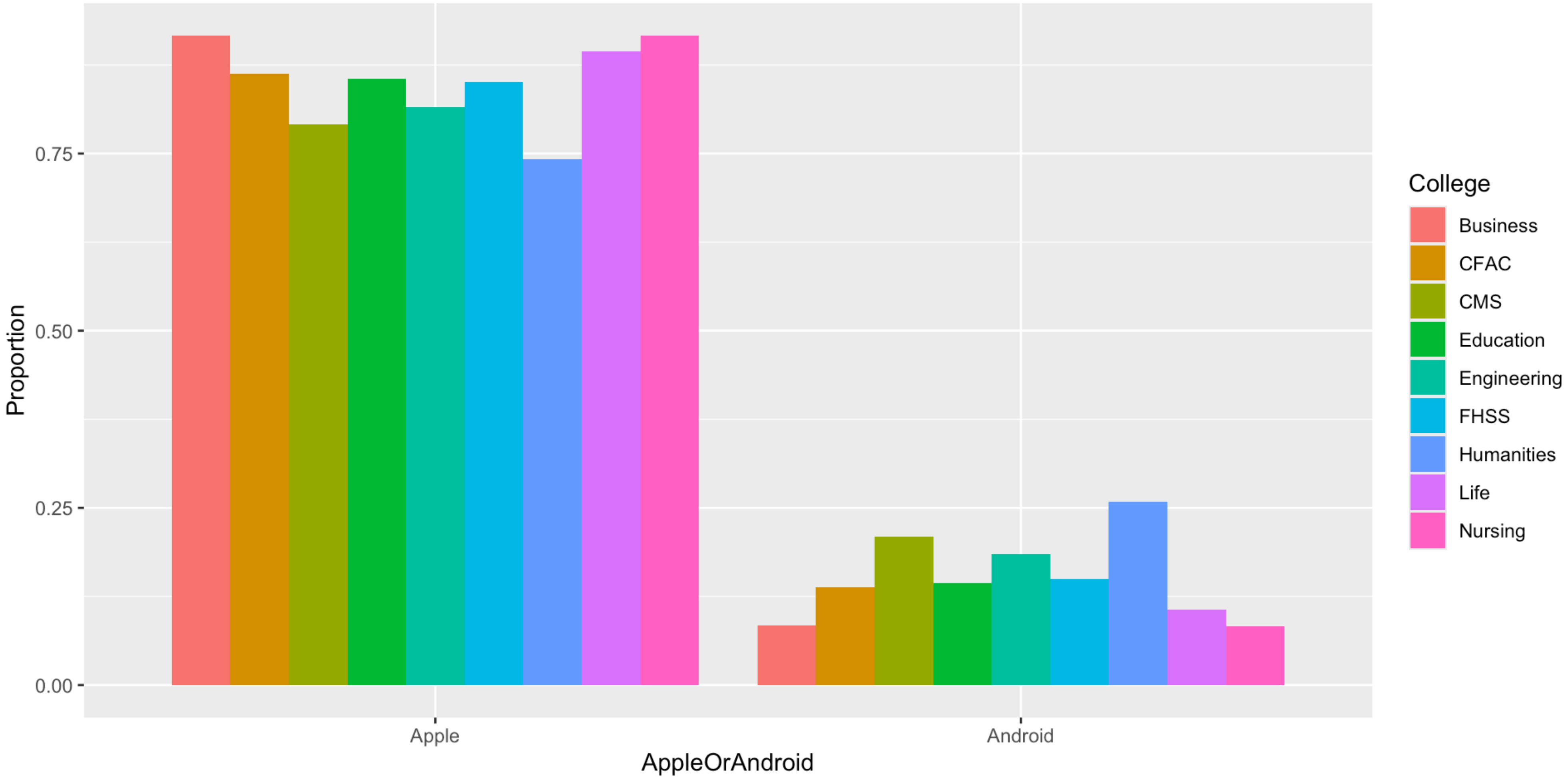
Main goal: Examine the RELATIONSHIP between College and Phone.

AppleOrAndroid	College
Apple	Business
Apple	CFAC
Apple	Business
Apple	CMS
Apple	Engineering

EDA Tool #1 - Grouped Bar Charts



EDA Tool #2 - Grouped Bar Charts Proportions



EDA Tool #3 - Tables of Counts

	Apple	Android	Sum
Business	557	51	608
CFAC	119	19	138
CMS	140	37	177
Education	95	16	111
Engineering	84	19	103
FHSS	91	16	107
Humanities	23	8	31
Life	329	39	368
Nursing	77	7	84
Sum	1515	212	1727

EDA Tool #4 - Conditional and Marginal Distributions

Main Idea: Convert counts to proportions to account for differences in count sizes

Conditional Distribution of Row Variable given Column Variable:

- proportions sum to 1 down the rows
- divide cell counts by column totals

Conditional Distribution of Column Variable given Row Variable:

- proportions sum to 1 across the columns
- divide cell counts by row totals

EDA Tool #4 - Conditional and Marginal Distributions

Marginal Distribution of Column (or Row)

- proportions sum to 1 across total column (or row)
- divide column (or row) totals by table total

Relationship between variables is probably present if conditionals are different than marginal distributions.

Cond. Dists of Col. Given Row

	Apple	Android
Business	0.916	0.084
CFAC	0.862	0.138
CMS	0.791	0.209
Education	0.856	0.144
Engineering	0.816	0.184
FHSS	0.850	0.150
Humanities	0.742	0.258
Life	0.894	0.106
Nursing	0.917	0.083
Margin (Overall)	0.877	0.123

Cond. Dists of Row Given Col.

	Apple	Android	Margin (Overall)
Business	0.368	0.241	0.352
CFAC	0.079	0.090	0.080
CMS	0.092	0.175	0.102
Education	0.063	0.075	0.064
Engineering	0.055	0.090	0.060
FHSS	0.060	0.075	0.062
Humanities	0.015	0.038	0.018
Life	0.217	0.184	0.213
Nursing	0.051	0.033	0.049

Practice: Age vs. Distracted Driving

	15-19	20-29	30-39	40-49	50-59	60-69	70+	Sum
Cell Phone Distracted	58	159	96	69	48	21	5	456
Not Distracted	2962	11278	8382	7328	7482	5282	4341	47055
Other Distracted	303	898	586	400	415	288	282	3172
Sum	3323	12335	9064	7797	7945	5591	4628	50683

Of those cell phone distracted drivers, what proportion are 15-19?

- 58/456

Is this a conditional or marginal proportion?

- Conditional

Practice: Age vs. Distracted Driving

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What is the conditional distribution of age for those who are cell phone distracted?

Practice: Age vs. Distracted Driving

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What is the conditional distribution of age for those who are cell phone distracted?

	15-19	20-29	30-39	40-49	50-59	60-69	70+	Sum
Cell Phone Distracted	0.127	0.349	0.211	0.151	0.105	0.046	0.011	1

Practice: Age vs. Distracted Driving

	15-19	20-29	30-39	40-49	50-59	60-69	70+	Sum
Cell Phone Distracted	58	159	96	69	48	21	5	456
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What is the conditional distribution of age for those who are cell phone distracted?

	15-19	20-29	30-39	40-49	50-59	60-69	70+	Sum
Cell Phone Distracted	0.127	0.349	0.211	0.151	0.105	0.046	0.011	1
Not Distracted	0.063	0.240	0.178	0.156	0.159	0.112	0.092	1
Other Distracted	0.096	0.283	0.185	0.126	0.131	0.091	0.089	1
Margin (Overall)	0.066	0.243	0.179	0.154	0.157	0.110	0.091	1

Practice: Age vs. Distracted Driving

	15-19	20-29	30-39	40-49	50-59	60-69	70+	Sum
Cell Phone Distracted	58	159	96	69	48	21	5	456
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What is the conditional distribution of distracted for those aged 20-29?

Practice: Age vs. Distracted Driving

	15-19	20-29	30-39	40-49	50-59	60-69	70+	Sum
Cell Phone Distracted	58	159	96	69	48	21	5	456
Not Distracted	2962	11278	8382	7328	7482	5282	4341	47055
Other Distracted	303	898	586	400	415	288	282	3172
Sum	3323	12335	9064	7797	7945	5591	4628	50683

What is the conditional distribution of distracted for those aged 20-29?

	20-29
Cell Phone Distracted	0.013
Not Distracted	0.914
Other Distracted	0.073
Sum	1.000

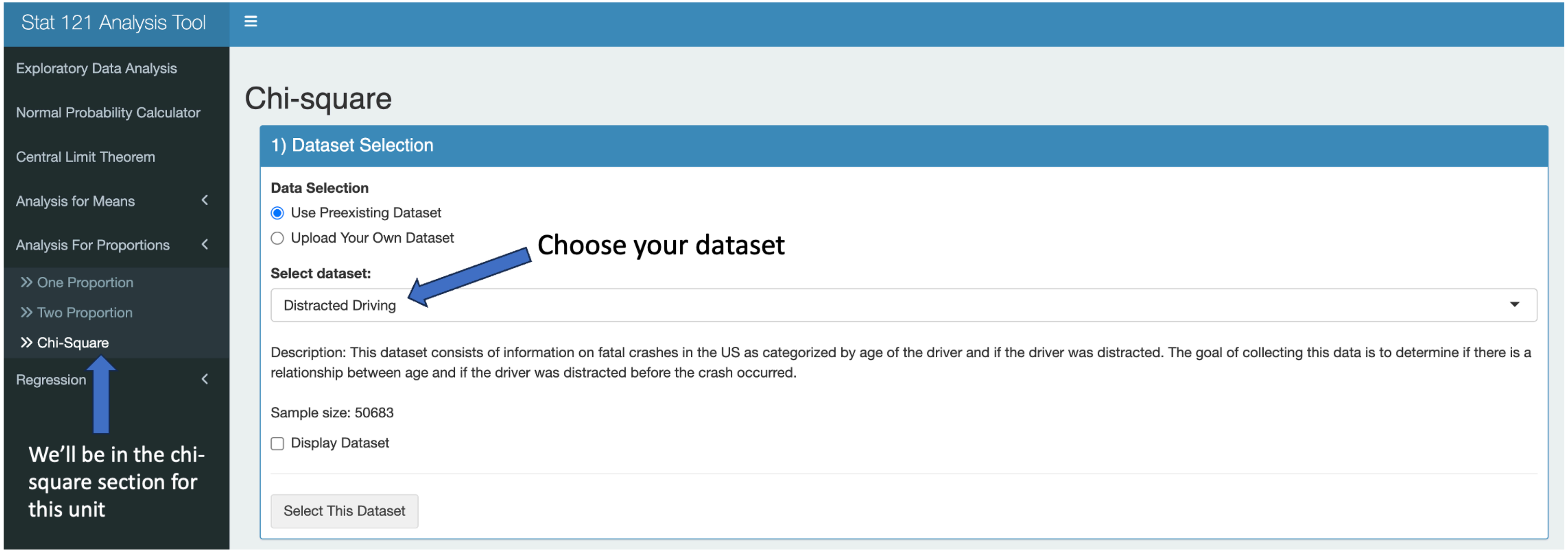
Practice: Age vs. Distracted Driving

	15-19	20-29	30-39	40-49	50-59	60-69	70+	Sum
Cell Phone Distracted	58	159	96	69	48	21	5	456
Not Distracted	2962	11278	8382	7328	7482	5282	4341	47055
Other Distracted	303	898	586	400	415	288	282	3172
Sum	3323	12335	9064	7797	7945	5591	4628	50683

What is the conditional distribution of distracted for those aged 20-29?

	15-19	20-29	30-39	40-49	50-59	60-69	70+	Margin (Overall)
Cell Phone Distracted	0.017	0.013	0.011	0.009	0.006	0.004	0.001	0.009
Not Distracted	0.891	0.914	0.925	0.940	0.942	0.945	0.938	0.928
Other Distracted	0.091	0.073	0.065	0.051	0.052	0.052	0.061	0.063

Using the Analysis Tool



Chi-square

1) Dataset Selection

Data Selection

● Use Preexisting Dataset

○ Upload Your Own Dataset

Select dataset: Distracted Driving

Description: This dataset consists of information on fatal crashes in the US as categorized by age of the driver and if the driver was distracted. The goal of collecting this data is to determine if there is a relationship between age and if the driver was distracted before the crash occurred.

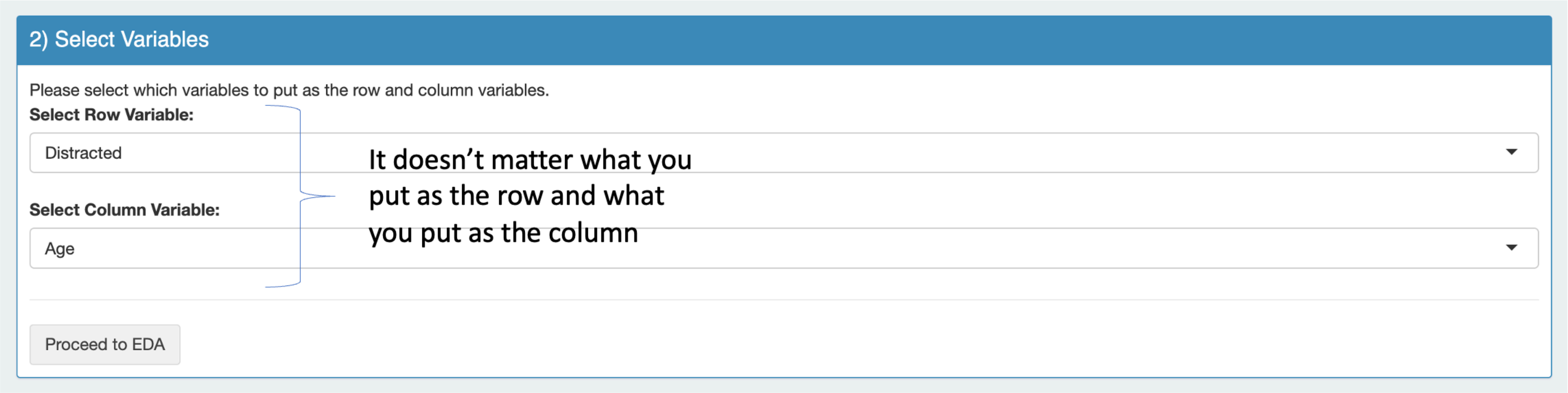
Sample size: 50683

□ Display Dataset

Choose your dataset

We'll be in the chi-square section for this unit

Using the Analysis Tool



2) Select Variables

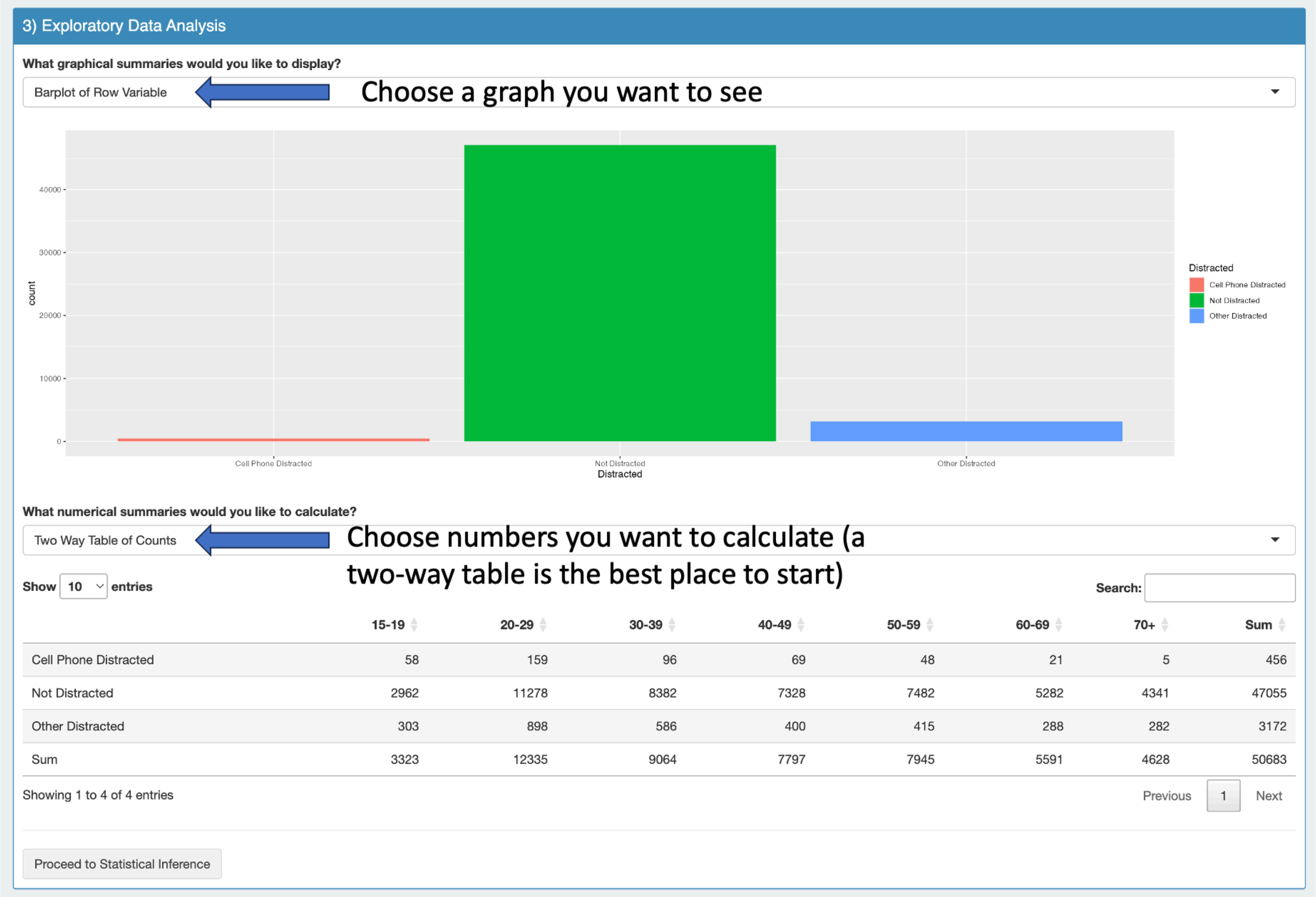
Please select which variables to put as the row and column variables.

Select Row Variable: Distracted

Select Column Variable: Age

It doesn't matter what you put as the row and what you put as the column

Using the Analysis Tool



3) Exploratory Data Analysis

What graphical summaries would you like to display?

Barplot of Row Variable

Choose a graph you want to see

What numerical summaries would you like to calculate?

Two Way Table of Counts

Choose numbers you want to calculate (a two-way table is the best place to start)

Show 10 **entries**

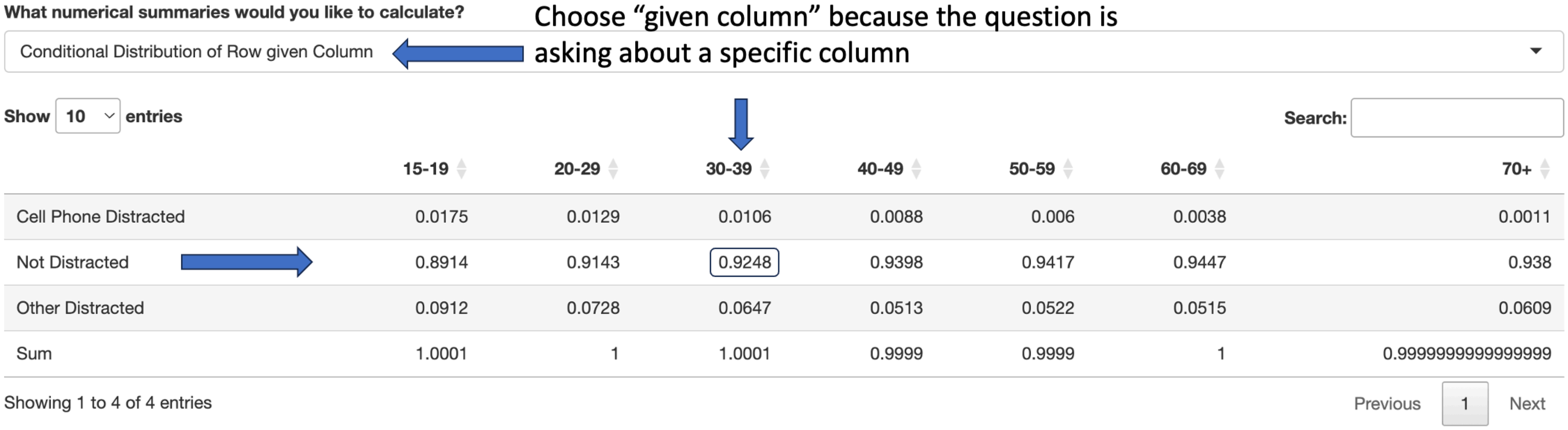
Search:

	15-19	20-29	30-39	40-49	50-59	60-69	70+	Sum
Cell Phone Distracted	58	159	96	69	48	21	5	456
Not Distracted	2962	11278	8382	7328	7482	5282	4341	47055
Other Distracted	303	898	586	400	415	288	282	3172
Sum	3323	12335	9064	7797	7945	5591	4628	50683

Showing 1 to 4 of 4 entries

Using the Analysis Tool

Of those drivers aged 30-39, what proportion are not distracted?



What numerical summaries would you like to calculate?

Conditional Distribution of Row given Column

Choose “given column” because the question is asking about a specific column

Show 10 entries

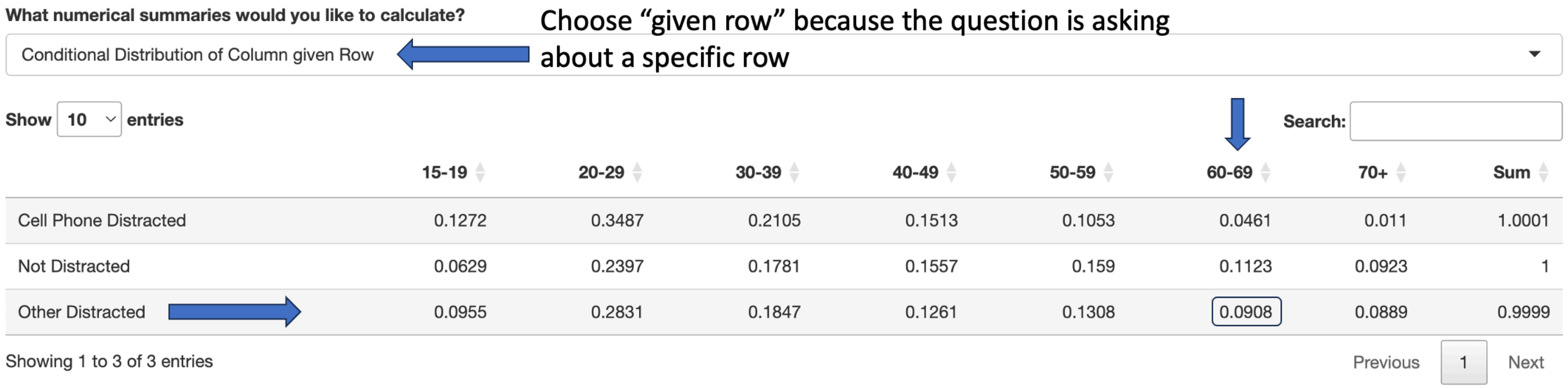
Search:

	15-19	20-29	30-39	40-49	50-59	60-69	70+
Cell Phone Distracted	0.0175	0.0129	0.0106	0.0088	0.006	0.0038	0.0011
Not Distracted	0.8914	0.9143	0.9248	0.9398	0.9417	0.9447	0.938
Other Distracted	0.0912	0.0728	0.0647	0.0513	0.0522	0.0515	0.0609
Sum	1.0001	1	1.0001	0.9999	0.9999	1	0.9999999999999999

Showing 1 to 4 of 4 entries

Using the Analysis Tool

Of those "other distracted" drivers, what proportion are 60-69?



What numerical summaries would you like to calculate?

Conditional Distribution of Column given Row

Choose "given row" because the question is asking about a specific row

Show 10 **entries**

	15-19	20-29	30-39	40-49	50-59	60-69	70+	Sum
Cell Phone Distracted	0.1272	0.3487	0.2105	0.1513	0.1053	0.0461	0.011	1.0001
Not Distracted	0.0629	0.2397	0.1781	0.1557	0.159	0.1123	0.0923	1
Other Distracted	0.0955	0.2831	0.1847	0.1261	0.1308	0.0908	0.0889	0.9999

Showing 1 to 3 of 3 entries

Statistical Model (Population)

The independence population model: The choice of apple vs. android product for a student is independent of the college of the student. In other words, the two variables are independent of each other.

Back to the Phone Example

	Apple	Android	Sum
Business	557	51	608
CFAC	119	19	138
CMS	140	37	177
Education	95	16	111
Engineering	84	19	103
FHSS	91	16	107
Humanities	23	8	31
Life	329	39	368
Nursing	77	7	84
Sum	1515	212	1727

Consequences of Independent Population Model

1. Because of independence…

$$\begin{aligned}\text{Pr(Apple \& Business)} &= \text{Pr(Apple)Pr(Business)} \\ &= (1515/1727) \times (608/1727) \\ &= 0.309\end{aligned}$$

2. IF variables are independent, expected number of people in each cell:

$$\begin{aligned}\text{Exp. No. of Apple/Business} &= n \times \text{Pr(Apple)Pr} \times \text{(Business)} \\ &= 1727 \times 0.309 \\ &= 533.364\end{aligned}$$

Independence Model Practice

	15-19	20-29	30-39	40-49	50-59	60-69	70+	Sum
Cell Phone Distracted	58	159	96	69	48	21	5	456
Not Distracted	2962	11278	8382	7328	7482	5282	4341	47055
Other Distracted	303	898	586	400	415	288	282	3172
Sum	3323	12335	9064	7797	7945	5591	4628	50683

1. Under the independence model, what is the probability of being 15-19 and not distracted?

- $(3323/50683) \times (47055/50683) = 0.061$

Independence Model Practice

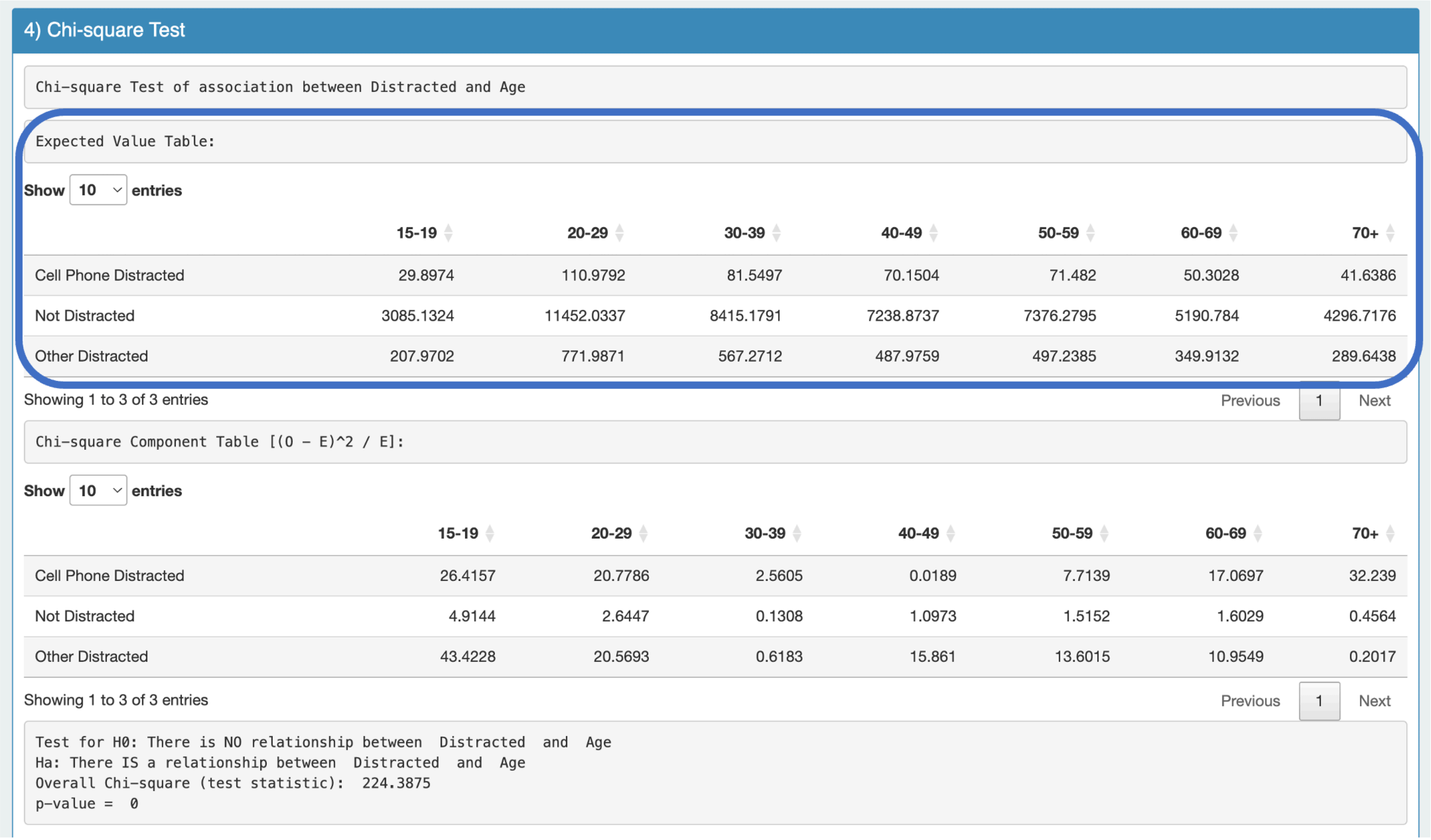
	15-19	20-29	30-39	40-49	50-59	60-69	70+	Sum
Cell Phone Distracted	58	159	96	69	48	21	5	456
Not Distracted	2962	11278	8382	7328	7482	5282	4341	47055
Other Distracted	303	898	586	400	415	288	282	3172
Sum	3323	12335	9064	7797	7945	5591	4628	50683

2. Under the independence model, what is the expected number of 15-19 year old drivers who are not distracted?

- $0.061 \times 50683 = 3085.132$

Using the Analysis Tool

Good news! The tool will calculate the expected counts for you. You just need to know where to look…



4) Chi-square Test

```
Chi-square Test of association between Distracted and Age
```

```
Expected Value Table:
```

	15-19	20-29	30-39	40-49	50-59	60-69	70+
Cell Phone Distracted	29.8974	110.9792	81.5497	70.1504	71.482	50.3028	41.6386
Not Distracted	3085.1324	11452.0337	8415.1791	7238.8737	7376.2795	5190.784	4296.7176
Other Distracted	207.9702	771.9871	567.2712	487.9759	497.2385	349.9132	289.6438

Showing 1 to 3 of 3 entries

```
Chi-square Component Table [(O - E)^2 / E]:
```

	15-19	20-29	30-39	40-49	50-59	60-69	70+
Cell Phone Distracted	26.4157	20.7786	2.5605	0.0189	7.7139	17.0697	32.239
Not Distracted	4.9144	2.6447	0.1308	1.0973	1.5152	1.6029	0.4564
Other Distracted	43.4228	20.5693	0.6183	15.861	13.6015	10.9549	0.2017

Showing 1 to 3 of 3 entries

```
Test for H0: There is NO relationship between  Distracted  and  Age
Ha: There IS a relationship between  Distracted  and  Age
Overall Chi-square (test statistic):  224.3875
p-value =  0
```

Hypothesis Testing

Recall the 3 steps of hypothesis testing:

- Formulate hypotheses
- See if data matches (or doesn’t) match the hypotheses
- Draw a conclusions about the parameter

Hypothesis Testing

Research Question: Is there a relationship between which college someone is in and whether they use an apple or android phone?

The hypotheses:

$$\begin{aligned} H_0 &: \text{College and Phone are Independent} \\ H_a &: \text{College and Phone are NOT Independent} \end{aligned}$$

Hypothesis Testing

Step 2: See if the data matches the hypotheses.

How can we compare our observed data to hypotheses?

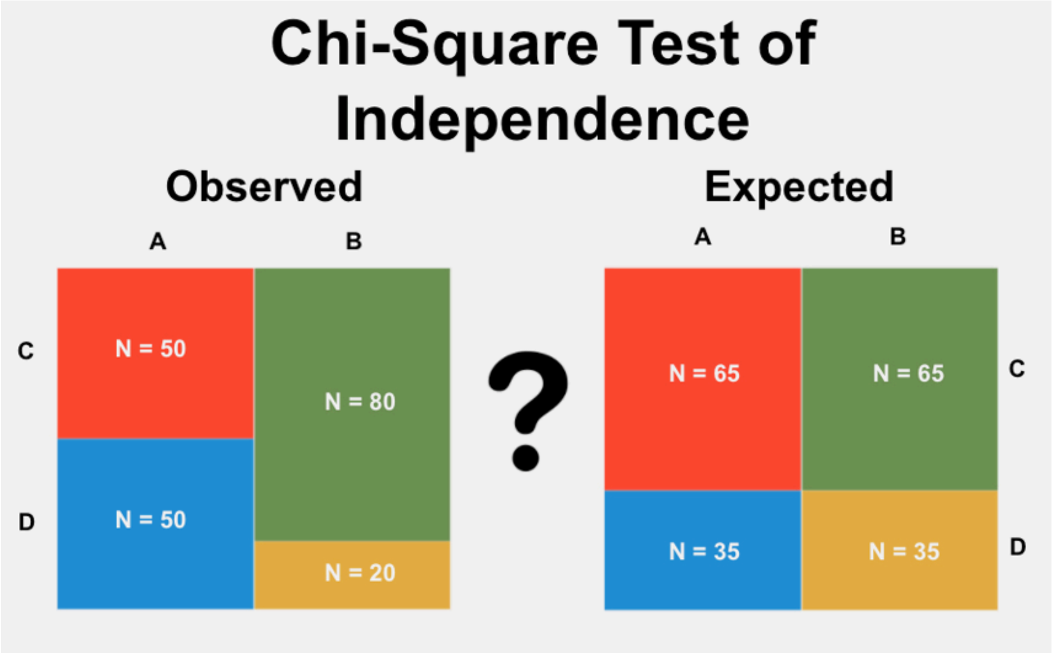
- Compare our data to what we expect to see IF the variables are independent.

Hypothesis Testing

Step 2: See if the data matches the hypotheses.

How can we compare our observed data to hypotheses?

- Compare our data to what we expect to see IF the variables are independent.



Hypothesis Testing

Step 2: See if the data matches the hypotheses.

How can we compare our observed data to hypotheses?

- Compare our data to what we expect to see IF the variables are independent.

The χ^2-statistic: (pronounced “kai-squared”)

$$\begin{aligned}\chi^2 &= \sum_{r=1}^{R}\sum_{c=1}^{C}\chi^2_{rc} \\ &= \sum_{r=1}^{R}\sum_{c=1}^{C}\frac{(\mathrm{Obs}_{rc}-\mathrm{Exp}_{rc})^2}{\mathrm{Exp}_{rc}}\end{aligned}$$

Hypothesis Testing

Step 2: See if the data matches the hypotheses.

$$\chi^2 = \sum_{r=1}^{R} \sum_{c=1}^{C} \frac{(\text{Obs}_{rc} - \text{Exp}_{rc})^2}{\text{Exp}_{rc}}$$

Intuition

- If χ^2 is big, then the data favor H_a because what you observed is different than what you expected to observe IF H_0 was true.
- If any individual cell χ^2_{rc} is big, then that observed count is very different from what you expected it to be if H_0 were true.

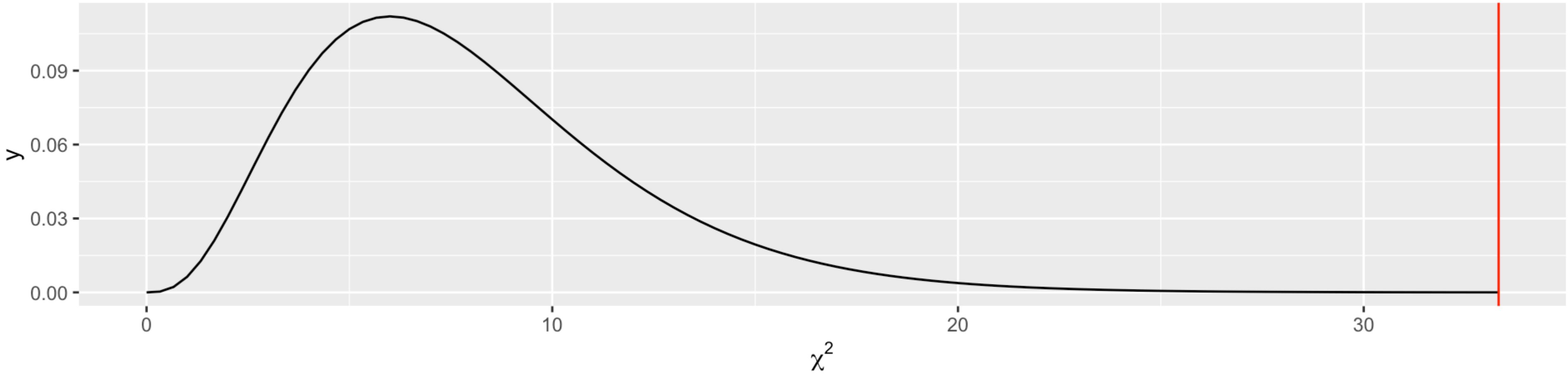
Hypothesis Testing

Step 2: See if the data matches the hypotheses.

Theorem. Sampling Distribution of Chi-2

If the independence model is appropriate AND all expected counts are > 5, then the χ^2 values that you should get when sampling follows an χ^2-distribution.

I am NOT going to get into details of what the χ^2 distribution is (it's technical) it looks like this.

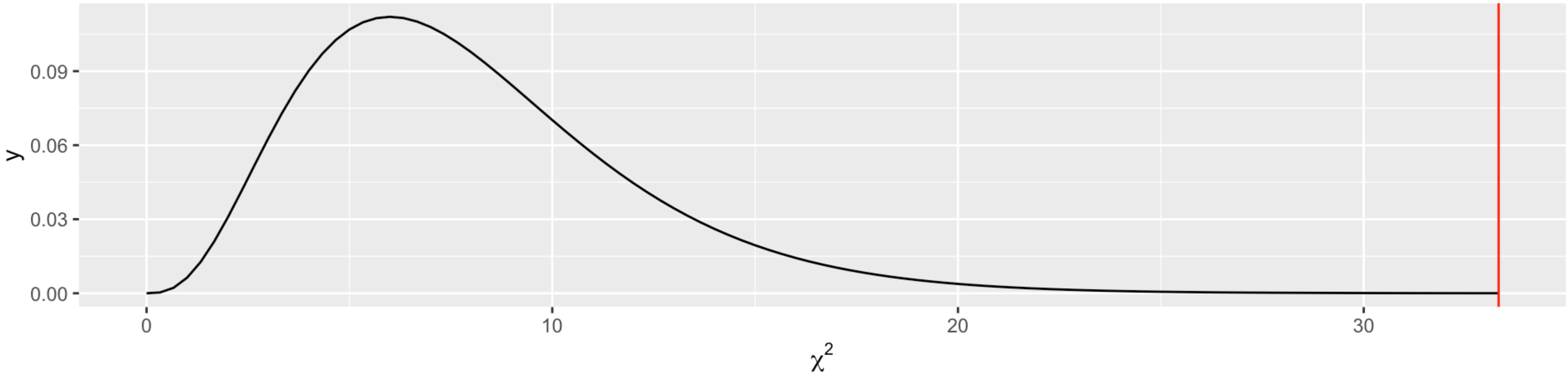


Hypothesis Testing

Step 2: See if the data matches the hypotheses (FIRST - check to make sure all expected counts > 5)

	Apple	Android
Business	533.364	74.636
CFAC	121.060	16.940
CMS	155.272	21.728
Education	97.374	13.626
Engineering	90.356	12.644
FHSS	93.865	13.135
Humanities	27.195	3.805
Life	322.826	45.174
Nursing	73.688	10.312

Hypothesis Testing



Step 2: See if the data matches the hypotheses.

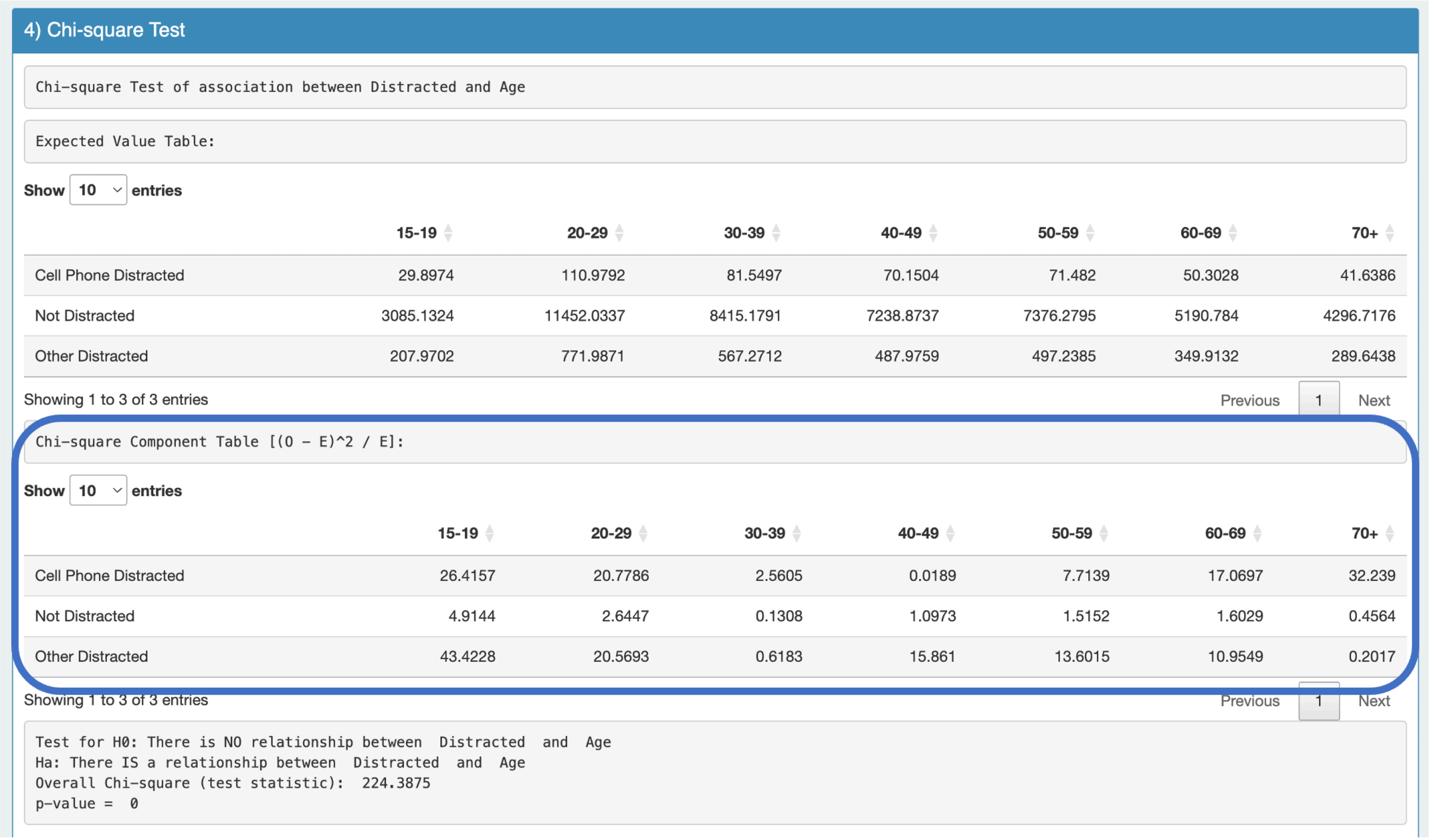
- $\chi^2 =$ 33.3255
- p-value $=$ 10^{-4}

What is your conclusion at the $\alpha = 0.05$ level?

- The data are inconsistent with the null hypothesis so we conclude that the college and phone variables are NOT independent.

Using the Analysis Tool

The tool calculates the χ^2_{rc} values for you:



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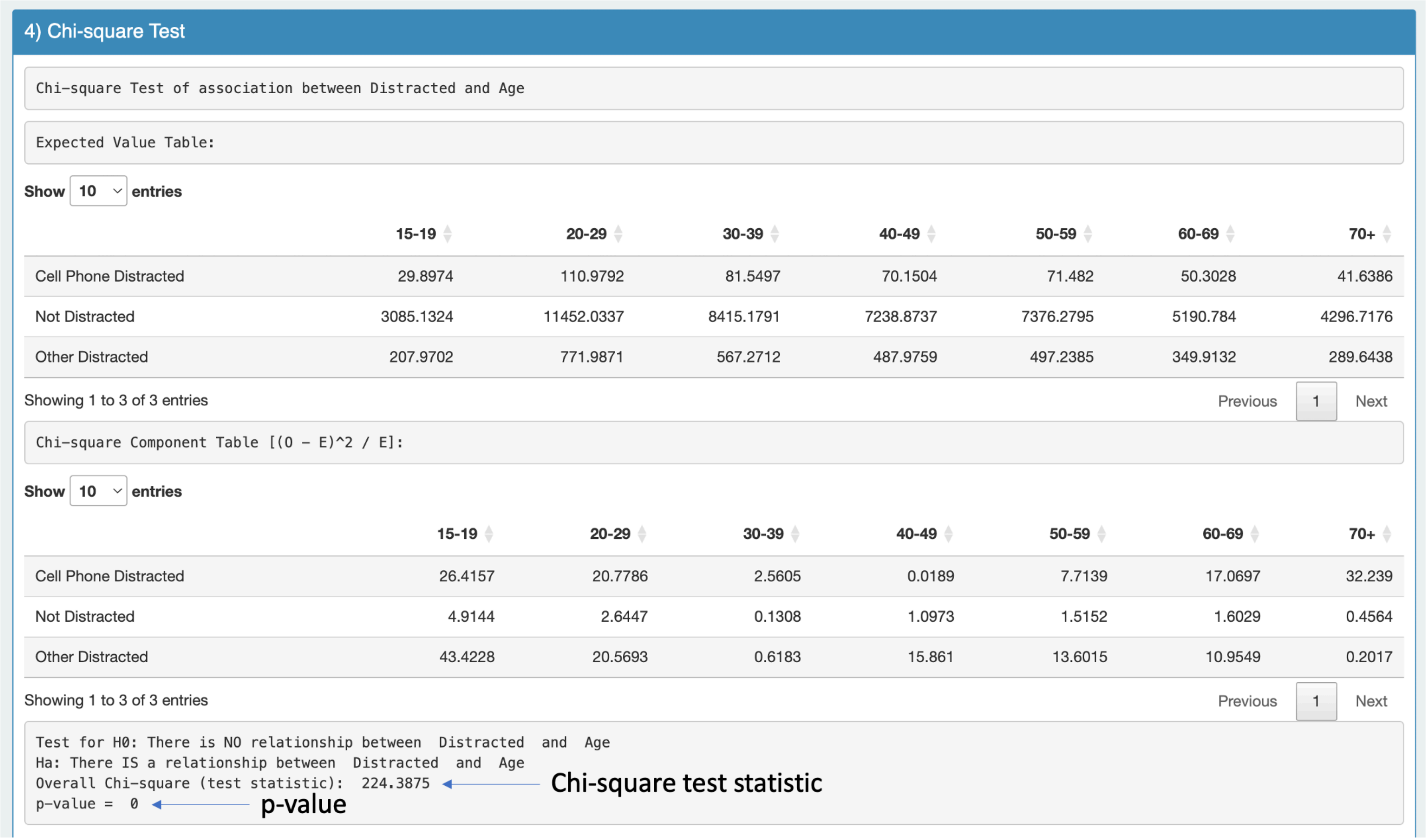
Using the Analysis Tool



4) Chi-square Test

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Chi-square Test of association between Distracted and Age
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Expected Value Table:
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Show 10 entries

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Chi-square test statistic

p-value

Following up on χ^2 Test

IF you reject H_0, what can we say about where the relationship is? In other words, where are observed counts most different from expected counts?

- Check the individual cell χ^2 values.

Following up on χ^2 Test

Chi-square Vals

	Apple	Andr
Business	1.0	
CFAC	0.0	
CMS	1.5	1
Education	0.1	
Engineering	0.4	
FHSS	0.1	
Humanities	0.6	
Life	0.1	
Nursing	0.1	

Obs. Counts

	Apple	Andr
Business	557	
CFAC	119	
CMS	140	
Education	95	
Engineering	84	
FHSS	91	
Humanities	23	
Life	329	
Nursing	77	

Exp. Counts

	Apple	Andr
Business	533.4	7
CFAC	121.1	1
CMS	155.3	2
Education	97.4	1
Engineering	90.4	1
FHSS	93.9	1
Humanities	27.2	
Life	322.8	4
Nursing	73.7	1

Nuances of χ^2 Tests

1. What do we do if our expected counts aren’t all > 5?

- Go get more data, combined small count categories or ask a statistician.

Key Terminology

- Conditional distributions
- Marginal Distributions
- Side-by-side bar charts
- Chi-square test
- Chi-square statistics